

Vanishing Phase Stiffness and Fluctuation-Dominated Superconductivity in UTe_2

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Superconductivity in three dimensions is almost universally governed by Ginzburg-Landau mean-field theory, with critical fluctuations typically confined to within a few percent of the transition temperature (T_c). We report that the heavy-fermion superconductor UTe_2 exhibits superconducting fluctuations that extend over a temperature range as wide as T_c itself—the largest observed for any three-dimensional superconductor. Through ultrasound measurements of the elastic moduli and sound attenuation, we find that UTe_2 transitions from a mean-field-like state at ambient pressure to a fluctuation-dominated state at higher pressures. This regime is marked by elastic softening and an increase in sound attenuation that onsets well above T_c , with the attenuation remaining anomalously high deep in the superconducting state. Our analysis suggests that these features stem from an extremely low superfluid phase stiffness. This results in a kinetic inductance as high as that of granular aluminum, but achieved in the clean limit. We propose a model where this exotic state is driven by dominant interband pairing mediated by ferromagnetic fluctuations, leading to “local” Cooper pairs with a coherence length of only a few lattice constants.

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I. INTRODUCTION

UTe_2 is a heavy-fermion superconductor that hosts two distinct superconducting phases under hydrostatic pressure [1], as well as three (or more) superconducting phases in an applied magnetic field [2–5]. The high upper critical fields of these phases suggest that spin-triplet pairing is likely involved, but the order parameter is still under debate [6,7], leaving the pairing mechanism(s) still very much an open question.

Similarities in the field-temperature phase diagrams of UTe_2 and the ferromagnetic superconductors UCoGe [8] and URhGe [9] have led to speculation that ferromagnetic fluctuations might play an important role in pairing. While such fluctuations have been observed in high magnetic fields in UTe_2 [10], the zero-field, ambient-pressure susceptibility is instead peaked at an antiferromagnetic wave vector [11,12]. Despite this, the fact that the high-pressure phase and the high field SC2 phase are smoothly connected

in the field-temperature-pressure phase diagram is highly suggestive that the pairing mechanism in these two phases is related [13]. In addition, first-principles calculations suggest that an intra-uranium-dimer ferromagnetic interaction is important for superconductivity for some region of parameter space [14], which may be realized under high if not ambient pressures.

Whatever the (likely unconventional) pairing mechanism, the phenomenology of UTe_2 is clearly well described by Ginzburg-Landau (GL) mean-field theory, at least at ambient pressure. For example, the specific heat anomaly at T_c is sharp, with no signature of fluctuation effects. The applicability of mean-field theory implies a strong superconducting phase stiffness, which is common among bulk metallic superconductors. A large phase stiffness may not, however, be universal to all superconducting phases of UTe_2 . For example, the fact that the SC2 transition line changes slope dramatically at the SC1 phase boundary [15], while the slope of the SC1 phase boundary continues uninterrupted, suggests that SC2 is in some way less robust than SC1 (see Fig. 1). This is consistent with magnetic susceptibility measurements that suggest a larger London penetration depth (implying a smaller phase stiffness) in the SC2 phase [16]. Any qualitative differences between the SC1 and SC2 phases that can be pinned down experimentally will be essential to sorting out the pairing mechanism,

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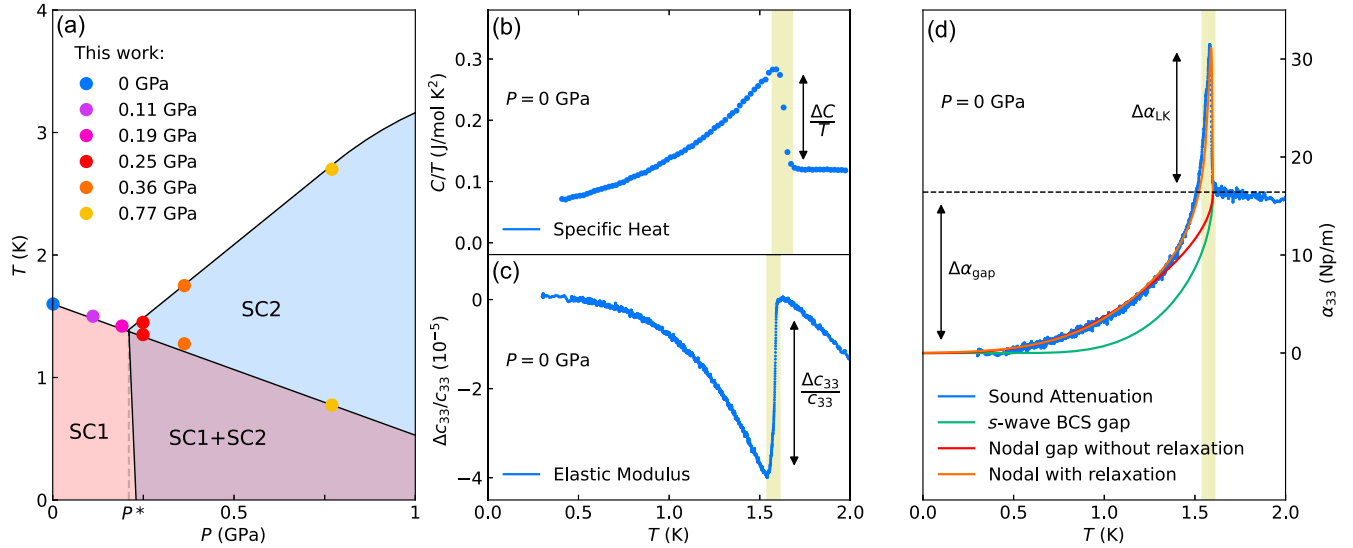


FIG. 1. (a) The temperature-pressure phase diagram of UTe_2 in zero magnetic field. Phase boundaries (black lines) are guides to the eye adapted from Braithwaite *et al.* [1] and Kamat *et al.* [15]. The two superconducting phases, SC1 and SC2, are shaded red and blue, respectively. Colored dots indicate transitions measured in the present work, and the pressure above which we observe two transitions is denoted by P^* (P^* is indicated by the dashed vertical line). (b) Specific heat of UTe_2 at ambient pressure as a function of temperature from Ref. [17]. A sharp upward jump in C/T is observed at T_{c1} upon entering the SC1 state. (c) Ambient-pressure c_{33} elastic modulus as a function of temperature. A sharp downward jump $\Delta C_{33}/C_{33}$ is seen at T_{c1} . (d) The longitudinal sound attenuation α_{33} at ambient pressure as a function of temperature. An order-parameter relaxation peak $\Delta\alpha_{LK}$ is seen just below T_c . As the quasiparticle density of states is gapped out below T_c , we see the conventional decrease $\Delta\alpha_{gap}$ in the attenuation. Fits to different models of gap attenuation are shown.

but to date there have been very few techniques applied under high pressures.

Here, we perform ultrasonic measurements of the c_{33} elastic modulus and the corresponding sound attenuation coefficient, α_{33} , on UTe_2 under pressure. We measure from ambient pressure up to ≈ 0.8 GPa and find that c_{33} evolves from exhibiting a sharp thermodynamic discontinuity in the form of a “jump” in c_{33} at T_c —characteristic of a mean-field superconducting transition—to exhibiting softening of the elastic modulus over a broad temperature range above and below T_c . This behavior suggests that UTe_2 evolves from a rather conventional mean-field superconductor at ambient pressure to a superconductor that is not well described by Ginzburg-Landau mean-field theory. The region over which we observe a breakdown of GL theory is larger than that of any known three-dimensional superconductor. In particular, we suggest that the SC2 phase is characterized by an anomalously low phase stiffness with “local” Cooper pairs whose coherence length is on the order of a few lattice constants. We introduce a simple model based on ferromagnetic fluctuations that leads to an interband pairing state with low phase stiffness.

II. EXPERIMENTAL METHODS

Single crystals of UTe_2 were grown with the chemical vapor transport method described in Ran *et al.* [17] and Ran *et al.* [4]. Two samples, S1 and S2, were cut from a single

bulk crystal for this study. The samples were aligned to better than 1° using x-ray diffraction performed in a Laue backscattering system. The crystals were then polished to produce two parallel faces normal to the (001) direction, and a 1- μm -thick ZnO transducer was fabricated using the method described in Theuss *et al.* [6]. The transducer excited ultrasonic waves with polarizations along the (100) and (001) directions, corresponding to shear (c_{55}) and longitudinal (c_{33}) elastic moduli, respectively. The longitudinal and shear signals were separated in the time domain using the known speeds of sound from Theuss *et al.* [18]. The relative changes in elastic moduli and the sound attenuation were determined using a traditional phase-comparison pulse echo technique (see Ref. [6]).

Samples were pressurized in a piston-cylinder pressure cell with a 5:1 mixture of methanol and ethanol as a pressure medium. This medium was chosen because it is relatively hydrostatic over the range of pressures we measure [19]. Our ultrasound transducers probe a small ($100 \times 100 \times 300 \mu\text{m}^3$) region of our sample, further decreasing the effects of any pressure gradients. The resistance of a manganin coil mounted inside the pressure cell was used to determine the pressure at room temperature. The pressure cell was heat sunk via a copper rod to the helium-3 pot of an Oxford Instruments Heliox insert which was then cooled in a variable temperature insert in an Oxford Instruments Teslatron dry magnet system. This setup enabled measurements from 300 K down to 300 mK.

At cryogenic temperatures, pressures were determined by comparing the change in T_c to existing measurements [1]. When more than one transition was measured, both T_{c1} and T_{c2} agreed well with existing data [1,13,20]. We measured at six pressures in total. The transitions are shown as colored dots in Fig. 1(a).

III. RESULTS

The elastic modulus c_{33} probes the sensitivity of the electronic free energy to compressional strain along the c axis— ϵ_{zz} —and was previously demonstrated to couple strongly to superconductivity in UTe_2 [6]. Our UTe_2 samples exhibit a single, sharp T_c of 1.6 K at ambient pressure [the main text shows data on sample S1 except at $P = 0.25$ GPa where we show data on S2; data on S2 at other pressures is shown in Supplemental Material (SM) [21]]. Figure 1(b) shows ambient-pressure ultrasound measurements on sample S1, and Fig. 1(c) shows specific heat measurements on a similarly grown sample.

Upon cooling through T_c , the condensation of an order parameter leads to a sharp upward jump $\Delta C/T$ in the specific heat. The c_{33} elastic modulus has a corresponding downward jump Δc_{33} that is proportional to the heat capacity jump through an Ehrenfest relation:

$$\Delta c_{33} = -(\partial T_c / \partial \epsilon_{zz})^2 \frac{\Delta C}{T_c}, \quad (1)$$

where $\partial T_c / \partial \epsilon_{zz}$ is the rate at which T_c varies with ϵ_{zz} strain. The sharp jumps in both C/T and c_{33} at T_c are expected for a mean-field-like phase transition.

The sound attenuation coefficient α_{33} probes how electronic quasiparticles relax when the Fermi surface is deformed by the strain ϵ_{zz} [29,30]. The most striking feature of the ambient-pressure data is the sharp peak in sound attenuation immediately below T_c [Fig. 1(d)]. While peaks in sound attenuation are not expected for conventional superconductors, similar peaks have been observed in UPt_3 [31] and UBe_{13} [32]. There, they were understood as coming from order-parameter relaxation due to a sign-changing superconducting gap [33]. A key characteristic of order-parameter relaxation is that the peak height should scale linearly with ultrasound frequency [33]. We demonstrate this linear-in-frequency dependence with a Lifshitz-Kosevich (LK) model of order parameter relaxation in Appendix B. This confirms that, while the peak is visually striking, it is only unconventional in the sense that it requires a sign-changing superconducting gap. Note that deep gap minima alone will not produce these peaks—the presence of these peaks upon entering both SC1 and SC2 constitute strong evidence that both states have sign-changing order parameters. Upon further cooling below T_c , the superconducting gap opens and suppresses the quasiparticle density of states, producing a drop in sound attenuation as expected for a superconductor.

Figure 2 shows the evolution of the sound attenuation and elastic modulus as a function of temperature at six

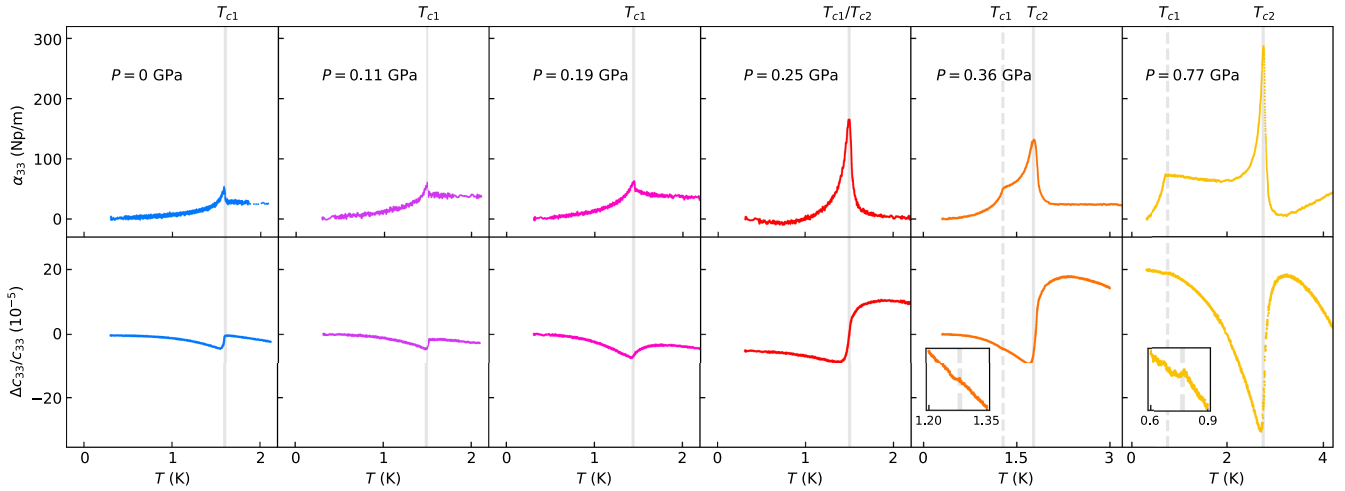


FIG. 2. Longitudinal sound attenuation α_{33} and fractional change in elastic modulus $\Delta c_{33}/c_{33}$ as a function of temperature at different pressures. T_{c1} and T_{c2} are indicated with gray lines. α_{33} exhibits a peak upon entering the SC1 state at the lowest three pressures ($P < P^*$). At the higher three pressures ($P > P^*$), the peak increases in magnitude and occurs upon entering the SC2 phase. The transition from the SC2 phase to the SC1 phase is clearly seen in the form of a kink as the attenuation decreases in the SC2 phase. Both features remain sharp at all pressures, indicating minimal pressure inhomogeneity (see SM [21] for a quantitative analysis). c_{33} exhibits a sharp drop upon entering the SC1 state at the lowest two pressures, as expected for a superconducting transition. We see fluctuations onset at pressures greater than 0.19 GPa, in the form of a decrease in c_{33} and increase in α_{33} , at temperatures greater than T_c . The size of the jump in c_{33} increases as the pressure is increased above P^* . The insets show a small jump that corresponds to the SC2-SC1 transition, which is seen at the highest two pressures. At the highest pressure of 0.77 GPa, the attenuation deep inside the SC2 superconducting state is larger than it is in the normal state, which is highly unusual for a superconductor.

different pressures spanning ambient to 0.77 GPa [the pressure values are shown in Fig. 1(a)]. At pressures greater than $P^* = 0.2$ GPa, the peak in α_{33} tracks the known phase boundary for SC2—we denote the transition to SC2 as T_{c2} , and the transition to SC1 as T_{c1} .

We first examine the evolution of sound attenuation with pressure. The peak in α_{33} is roughly constant in amplitude for pressures less than the critical pressure $P^* = 0.2$ GPa [1]. The peak then increases substantially in magnitude for pressures larger than P^* . At the highest two pressures, α_{33} drops upon entering the SC1 state at T_{c1} .

There are several unexpected features in this dataset. (1) For pressures near and above P^* ($P \geq 0.19$ GPa), α_{33} increases as T_c is approached in the normal state (i.e., from $T > T_c$). (2) At the highest pressure, the attenuation in the superconducting state is substantially larger than in the normal state, even deep inside SC2, only dropping in a conventional manner at T_{c1} . Note that the peak in α_{33} at T_{c1} remains sharp with increasing pressure for $P < P^*$, as does the drop in α_{33} at T_{c1} for $P > P^*$. This indicates that there is minimal broadening of the transitions due to pressure inhomogeneity.

Next, we examine the evolution of c_{33} with pressure. The jump in c_{33} is roughly constant in magnitude for pressures less than P^* . Crossing to pressures higher than P^* , the jump across the upper transition increases by a factor of 3. This corresponds to an increase in $(dT_c/d\epsilon_{zz})^2$ in Eq. (1), consistent with a more pressure-sensitive SC2 state with a larger slope of dT_c/dP . At the two highest pressures, where both transitions are well separated in temperature, we find a small jump in c_{33} at the lower transition (see insets of Fig. 2).

Concurrent with the increase in α_{33} at temperatures above T_c is the onset of softening in c_{33} . Like the increase in α_{33} above T_c , this softening is not expected for a mean-field phase transition, and is suggestive of new physics associated with SC2.

IV. ANALYSIS

A. Quantifying the fluctuation region

Broad fluctuation regimes, while rare in superconductors, are not uncommon for other phase transitions. For example, magnetic systems typically show large fluctuation regions due to diverging spatial correlations approaching the phase transition [34]. The softening that we observe above T_c in c_{33} , and corresponding increase in α_{33} , is therefore suggestive of spatial fluctuations in the magnitude and phase of the superconducting order parameter. To explore this idea, we extend the standard mean-field analysis to include small critical fluctuations of the order parameter. Here, we focus on modeling c_{33} because it is a thermodynamic quantity and therefore easier to evaluate than α_{33} , which is a transport quantity.

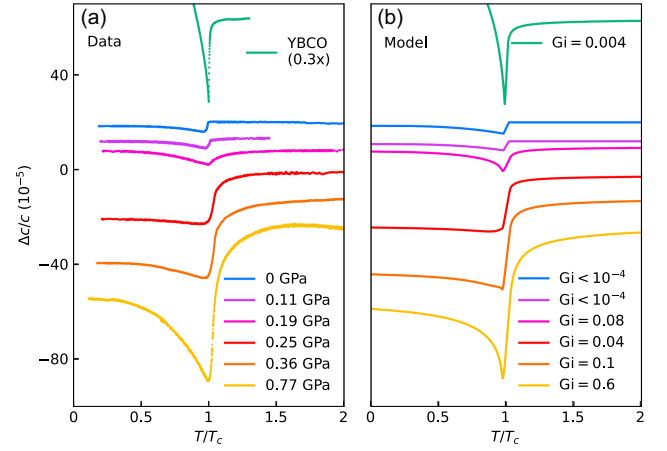


FIG. 3. (a) The elastic modulus c_{33} across the superconducting transition as a function of temperature, with the background subtracted out. We also show the elastic response of $\text{YBa}_2\text{Cu}_3\text{O}_7$ —a superconductor with a relatively large fluctuation region. Curves have been vertically offset for clarity. (b) Theoretical predictions for c_{33} using the Gaussian fluctuations model from the analysis section. The Ginzburg number Gi quantifies the strength of the fluctuations and increases at higher pressures.

First, to isolate the superconducting contribution to the temperature dependence of c_{33} , we use the ambient-pressure data for $T > T_c$ as a background. We find that this background models the high temperature behavior of c_{33} at all other pressures when scaled by a single pressure-dependent parameter. In SM [21], we show that this parameter is approximately the Kondo coherence scale T_K , which manifests as a minimum in the temperature dependence of c_{33} . We subtract the scaled background at each pressure and show the resulting curves in Fig. 3(a). For comparison, we also show data for $\text{YBa}_2\text{Cu}_3\text{O}_7$ —a superconductor with a relatively large fluctuation region (of order a few percent of T_c).

Second, we analyze the contribution to c_{33} with a model of Gaussian superconducting fluctuations. This model is intended to highlight the dependence of fluctuations on material parameters, such as phase stiffness, and is only a qualitative picture—Gaussian fluctuations will fail to describe the critical behavior close to T_c , where fluctuations are large.

The free energy of a superconductor, including contributions from a spatially varying order parameter $\psi(\vec{r})$, is

$$F = \int d\vec{r} \left[f_n + \alpha(T) |\psi|^2 + \frac{1}{2} \beta |\psi|^4 + K |\nabla \psi|^2 - \gamma \epsilon_{zz} |\psi|^2 \right], \quad (2)$$

where f_n is normal-state free-energy density including the elastic energy, $\alpha \equiv \alpha_0(T - T_c)$ and β are Landau coefficients, $K |\psi|^2 \equiv \kappa$ is the superfluid phase stiffness, and $\gamma \equiv \alpha_0(dT_c/d\epsilon_{zz})$ is the strain-superconductivity coupling

constant. Equation (2) can be solved exactly in the limit of small fluctuations of the order parameter. This yields Δc_{33} in the vicinity of T_c to be

$$\Delta c_{33} = -\left(\frac{dT_c}{d\epsilon_{zz}}\right)^2 \frac{\Delta C_{MF}}{T_c} \begin{cases} \sqrt{\frac{Gi}{2t}} & T > T_c \\ \left(1 + \sqrt{\frac{Gi}{t}}\right) & T < T_c. \end{cases} \quad (3)$$

Here, ΔC_{MF} is the mean-field heat capacity jump, $t = (T - T_c)/T_c$ is the reduced temperature, k_B is the Boltzmann constant, and Gi is the dimensionless Ginzburg number given by

$$Gi = \frac{\Delta C_{MF} T_c}{\kappa_0^3} \left(\frac{k_B T_c}{2\pi}\right)^2, \quad (4)$$

where κ_0 is the phase stiffness at zero temperature. In the limit of large phase stiffness, Gi vanishes and Eq. (3) reduces to equation (1); i.e., there is no fluctuation contribution to c_{33} when the phase stiffness is large. For a finite phase stiffness, a contribution that diverges as $1/\sqrt{t}$ turns on for both $T < T_c$ and $T > T_c$.

Mean-field theory fails for temperatures close to T_c given by $|T - T_c| < T_c \times Gi$, or $t < Gi$. In conventional superconductors, Gi is of order 10^{-8} and the fluctuation region is never observed. Experimentally, we define the fluctuation region as the temperature range over which the fluctuation contribution to the heat capacity and c_{33} is equal to its mean-field value, i.e., when $t = Gi$.

Figure 3(b) plots Eq. (3) with Gi adjusted to best match the experimental data (see SM for details on how the fits are performed [21]). For the two lowest pressures, we observe no precursor fluctuations and extract conventional Ginzburg numbers ($Gi < 10^{-4}$). Fluctuations onset at $P = 0.19$ GPa—slightly less than P^* (this is confirmed by a single transition in c_{55} ; see Appendix A). The fluctuations grow as the pressure is increased until the $t^{-\alpha}$ contribution dominates the signal near T_{c2} at $P = 0.77$ GPa.

Note that the quantitative values of Ginzburg numbers extracted in this way should not be taken too literally—the central point of this analysis is to show that the data at higher pressures are modeled by an increasing fluctuation contribution that diverges at T_c . Fits to a $t^{-1/2}$ power law obtained from Gaussian fluctuations are shown in SM [21]—the data fit qualitatively well, but the exact critical exponent may be different depending on the universality class of the transition. Critical fluctuations should also be observable in specific heat measurements, and recent quantitative specific heat measurements [35] exhibit fluctuations consistent with those measured in this work. For a detailed comparison, see Appendix F.

B. Model of superconductivity with low phase stiffness

Figure 3(a) shows a clear increase in the fluctuation contribution to c_{33} at pressures greater than P^* . This is

evident in the raw data, which show both softening in c_{33} and an enhancement of α_{33} above T_{c2} . The fluctuation contribution is quantified by the large Ginzburg number we extract in Fig. 3(b).

From where do the fluctuations originate? The two lowest pressures show no fluctuation region associated with SC1. At $P = 0.19$, fluctuations become apparent even though only SC1 is present—we attribute this to the close proximity of P^* at this pressure (see Appendix A). Above P^* , increasing pressure increases the strength of fluctuations at T_{c2} , while there are no fluctuations at the lower T_{c1} . Clearly the fluctuations are associated with the SC2 phase.

Why does the SC2 phase exhibit strong fluctuations of the order parameter while SC1 does not? At the level of the analysis above, this question becomes, why is Gi large in SC2 and small in SC1? Equation (4) suggests that a low superfluid phase stiffness in the SC2 phase could be responsible.

In a single-band BCS superconductor, the phase stiffness is simply the superfluid density divided by the quasiparticle effective mass. Naively, then, a low phase stiffness could be attributed to an anomalously small number of carriers, or to anomalously heavy carriers, condensing at T_{c2} . These explanations of our data face two difficulties: (1) the SC1 phase has an entirely conventional Gi , and there is no indication that the band structure of UTe_2 changes dramatically with pressure; and (2) the anomalies in C/T at T_{c1} and T_{c2} are within a factor of 2 of each other in size for pressures between $P = 0.38$ and 0.79 GPa [13], and therefore it cannot be that the mass (i.e., density of states) of the carriers condensing at T_{c2} is significantly different from those condensing at T_{c1} .

Instead, we suggest that *interband* pairing can produce an anomalously low phase stiffness. To demonstrate this, we consider pairing mediated by intradimer ferromagnetic fluctuations. Note that these fluctuations are distinct from the thermally driven superconducting fluctuations we see in our data. Such fluctuations have been suggested to be relevant by first-principle calculations [14] and are consistent with the dimer ferromagnetic configuration observed in spin fluctuations at ambient pressure [36] and in the magnetically ordered state at high pressure [37]. These fluctuations are known to support a pairing state that is spin-triplet and site-singlet [38] [i.e., the Cooper pair orbital wave function is $\Psi(r_1, r_2) = 1/\sqrt{2}[\phi_1(r_1)\phi_2(r_2) - \phi_2(r_1)\phi_1(r_2)]$, where ϕ_1 and ϕ_2 are the states at the two uranium sites, and r_1 and r_2 are the electronic coordinates]. When projected onto bands, this pairing state has both intraband and interband components because the two uranium states hybridize to form bonding and antibonding bands in the momentum basis. In most theoretical analyses of this pairing state [38–40] the intraband components dominate the pairing state. However, such a state would be expected to exhibit a usual BCS-like superfluid stiffness.

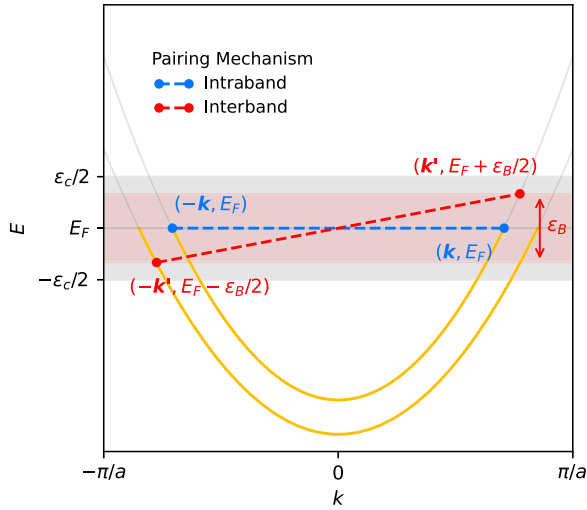


FIG. 4. An illustration depicting intraband and interband pairing for a simple two-band parabolic dispersion. Blue dots represent intraband pairs at momenta k and $-k$, and red dots represent interband pairs at momenta k' and $-k'$. The intraband pairs sit at the Fermi energy E_F . The gray shaded area represents the pairing energy shell. For an interband pairing state, this pairing energy (ϵ_c) must be greater than the band splitting ϵ_B [41], which is denoted by the red shaded area.

Recently, Samokhin showed [41] that a dominant interband pairing state can emerge as the superconducting state with the highest T_c if the following two conditions are satisfied: (i) the interaction for the interband pairing component is the largest and (ii) the interband component is Josephson coupled to the intraband components. Conditions (i) and (ii) are naturally satisfied from interdimer ferromagnetic fluctuations, where the relative weight of intraband to interband pairing is given by the ratio of intradimer to interdimer hopping ϵ . When $\epsilon \ll 1$, the intraband pairing interaction is a factor of ϵ^2 smaller than the interband pairing interaction. Taking the intradimer and interdimer hopping parameters from Ref. [6], where they were fitted to reproduce the Fermi surface measured by quantum oscillations, we estimate $\epsilon^2 \approx 6 \times 10^{-3}$. This suggests that the interband pairing state could be dominant and produce a stable pairing state under favorable circumstances.

This dominant interband pairing state has a superfluid stiffness that is much smaller than the more usual intraband pairing states. This is because, for uniform superconducting states where $\vec{k}$ and $-\vec{k}$ are paired, conventional intraband pairs are formed at the Fermi energy, whereas interband pairs are necessarily formed at a finite energy difference (Fig. 4). Generically, then, the phase stiffness of the interband state is small compared to the phase stiffness of the intraband state by the larger of two possible factors. The first factor is $(k_B T_c / \mathcal{E}_b)^2$, where $\mathcal{E}_b$ is the energy splitting between the two bands, which is the intrinsic

phase stiffness of the interband pairing state. The second factor is ϵ^2 , which emerges from the weak Josephson-coupled intraband pairing contributions (see Appendix C for more details). In our case, T_c is smaller than the interdimer hopping, so that the factor ϵ^2 determines the phase stiffness. Again taking $\epsilon^2 \approx 6 \times 10^{-3}$ from Theuss *et al.* [6], we obtain a Ginzburg number for the interband state of $\text{Gi}_{\text{inter}} \approx (5 \times 10^6) \text{Gi}_{\text{intra}}$. This is consistent with the (minimum) 4-order-of-magnitude increase in Gi we observe when transitioning from SC1 to SC2 (independent estimates of Gi at ambient pressure give $\text{Gi} \approx 10^{-5}$; see SM [21]). It is worth noting that the increase in Gi we observe with increasing pressure across the SC2 phase is consistent with the intradimer and interdimer hopping ratios ϵ , changing by a factor of 2.4, which is not unreasonable.

V. DISCUSSION

Given the above analysis, we suggest that the primary distinguishing feature between SC1 and SC2 is that SC1 is an intraband-paired superconductor, with a conventional, BCS-like phase stiffness, whereas SC2 is an interband-paired superconductor, with an anomalously low phase stiffness. This is consistent with the suggestion of an anomalously long penetration depth in the SC2 phase under pressure [16] and other works [42–45] that suggest different pairing mechanisms in the SC1 and SC2 phases. Note that this does *not* imply a low density of Cooper pairs. Instead, the BCS-GL expression relating phase stiffness and superfluid density is no longer applicable in the interband state.

A large fluctuation region has direct implications for the spatial extent of the Cooper pairs, independent of the pairing mechanism. Intuition for this can be gained by examining magnetic phase transitions, where large Ginzburg numbers are not uncommon: $\text{Gi} \approx 0.14$ in YbRh_2Si_2 , for example [46]. This departure from mean-field behavior is associated with the short length scale of the magnetic exchange interaction (typically nearest or next-nearest neighbor), allowing for large spatial fluctuations of the order parameter. In superconductors, the relevant length scale is the coherence length, which is typically thousands of lattice sites. Using the GL definitions of phase stiffness and coherence length, a Ginzburg number of 0.6 in UTe_2 at 0.77 GPa can be translated to a coherence length of only a few lattice constants: roughly 8 Å. This local Cooper pairing is in stark contrast with nearly all other superconductors, with the possible exception of underdoped cuprates [22]. A short coherence length is consistent with the unusually large upper critical field ($H_{c2} > 35$ T) of the SC2 state [2]. We also note that an extended fluctuation regime can enable the existence of exotic precursor nonsuperconducting phases, such as the putative fermion quadrupling phase observed in $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ [47]. To date we do not have any thermodynamic evidence for such a phase above T_{c2} .

To the best of our knowledge, the fluctuation regime shown in Fig. 3 at $P = 0.77$ GPa is larger than in any other three-dimensional superconductor, emphasizing the truly unique nature of the SC2 phase in UTe_2 . The behavior of the sound attenuation at this pressure is particularly striking—even below the sharp peak at T_{c2} , the attenuation remains significantly higher in the superconducting state as compared to the normal state. It will be interesting to probe this region of the phase diagram with other dynamic probes, such as microwave conductivity and NMR. As we demonstrate in Appendix V, the low phase stiffness of SC2 at $P = 0.77$ GPa translates to a kinetic inductance rivaling that of granular aluminum and NbTiN [48]. Stabilizing the SC2 phase using epitaxial strain could provide an interesting route to building high kinetic inductance superconducting elements in the clean limit.

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DATA AVAILABILITY

The data that support the findings of this article are openly available at Ref. [49].

APPENDIX A: TRANSVERSE-MODE c_{55} DATA

In addition to the c_{33} compressional elastic modulus shown in the main text, we have also measured a c_{55} shear modulus. Data at 0.11 and 0.19 GPa are measured on sample 1, 0.25 to 0.77 GPa are measured on sample 2, and the 0 GPa data are sample 3. These data are shown in Fig. 5. For a one-component order parameter we do not expect either a jump at T_c in the shear modulus [6] nor a Landau-Khalatnikov peak in the shear attenuation [33]. This is exactly what we see in the data of Fig. 5, with a sharp drop in the attenuation at T_{c1} and sharp kinks in the elastic modulus at both T_{c1} and T_{c2} (for pressures $P > P^*$). The sharp features at both T_c 's indicate minimal pressure broadening over the entire pressure-temperature space we measure.

The c_{55} data in Fig. 5 show that we measure a single transition at $P = 0.19$ GPa, indicating that the sample enters the SC1 phase only at this pressure. We attribute the fluctuations we observe in the c_{33} data at this pressure to its proximity to the SC2 phase. We illustrate this by showing a schematic of the (diverging) fluctuation contribution $c_{33,F}$ to c_{33} as a color plot on the P - T phase diagram (Fig. 6):

$$\Delta c_{33,F} = -\left(\frac{dT_c}{dc_{zz}}\right)^2 \frac{\Delta C_{MF}}{T_c} \begin{cases} \sqrt{\frac{G_i}{2t}} & T > T_c \\ \sqrt{\frac{G_i}{t}} & T < T_c \end{cases} \quad (\text{A1})$$

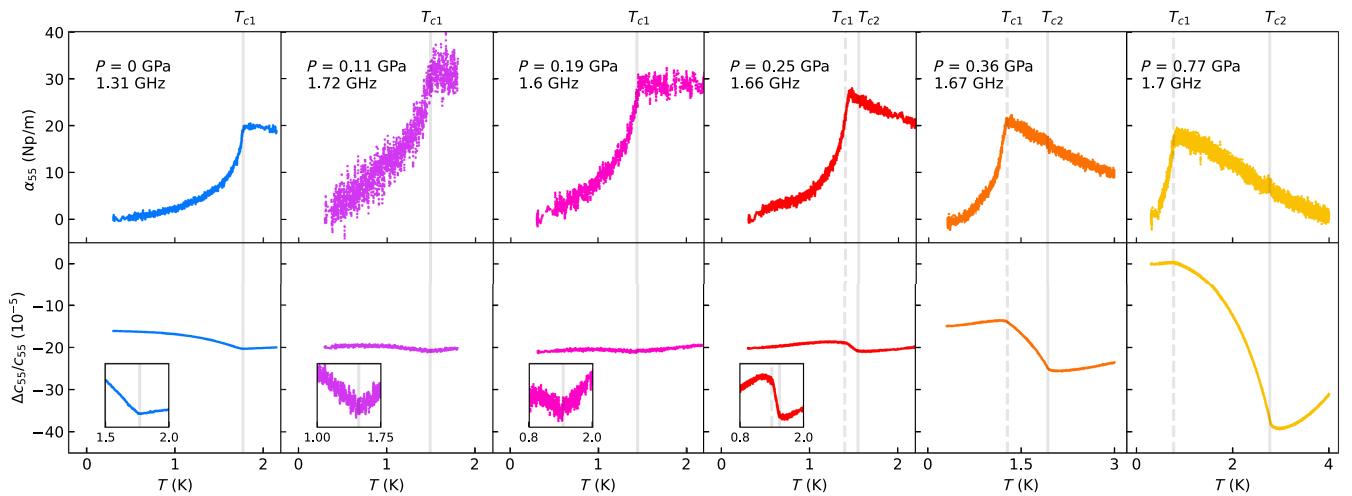


FIG. 5. Sound attenuation and the shear elastic modulus c_{55} . There is a sharp drop in the attenuation at T_{c1} at all pressures, following the conventional expectation of BCS theory. Sharp kinks are seen in the elastic modulus at both T_{c1} and T_{c2} . Insets show an enlarged elastic modulus, clearly showing two T_c 's for $P > P^*$.

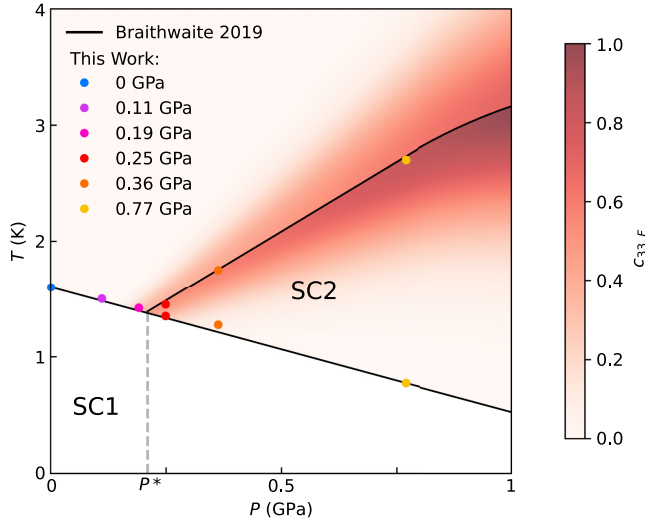


FIG. 6. A schematic diagram of the fluctuation contribution ($c_{33,F}$) to c_{33} overlaid on the UTe_2 phase diagram. Deeper red color indicates a larger fluctuation contribution. Colored triangles indicate T_c 's at the pressures at which we measure. Because of the absence of a phase boundary in the normal state, any thermodynamic variable (like $c_{33,F}$) cannot change discontinuously as pressure is varied in the normal state. This causes a “spillover” of the fluctuation contribution to pressures less than P^* , leading us to observe fluctuations at $P = 0.19$ GPa at temperatures just above T_c .

APPENDIX B: LANDAU-KHALATNIKOV DAMPING

In the main text, we attribute the peaks observed in α_{33} at T_c to order-parameter relaxation. Here, we construct a mean-field model of order-parameter relaxation and fit it to the 0 GPa data—where we have a broad frequency dependence of α_{33} —to show good quantitative agreement.

We begin by considering a Ginzburg-Landau free-energy density (f) expression in the presence of a strain ϵ . The coupling between the strain and the superconducting order parameter ψ is proportional to $\epsilon|\psi|^2$, which is the lowest order coupling between a strain and a superconducting order parameter that preserves gauge symmetry [6].

$$f = f_n + \alpha(T)|\psi|^2 + \frac{1}{2}\beta|\psi|^4 - \gamma\epsilon|\psi|^2 \quad (\text{B1})$$

$\alpha(T) \equiv \alpha_0(T - T_c)$ and β are the Ginzburg-Landau phenomenological parameters that model the symmetry breaking phase transition. γ sets the strength of the strain-order parameter coupling.

This coupling is only allowed when the product $\epsilon|\psi|^2$ is a scalar, and for an orthorhombic system like UTe_2 , can happen only for a longitudinal strain, or for a shear strain with a two-component order parameter (ruled out for the SC1 phase by Theuss *et al.* [6]).

In the mean-field approximation, we can find the equilibrium value of the order-parameter magnitude $|\psi_0|$

for a given strain and temperature by setting $\partial f / \partial \psi = 0$ and solving for ψ . If the order parameter is shifted away from this equilibrium value (e.g., by the application of strain), we expect it to return back to equilibrium. It is here that we make our first assumption about the dynamics of the system. A deviation of the order-parameter magnitude $|\psi| = |\psi_0| + |\delta\psi|$ from equilibrium decreases as

$$\frac{\partial(|\delta\psi|)}{\partial t} = -\xi \frac{\partial f}{\partial |\psi|}, \quad (\text{B2})$$

where ξ quantifies the “inertia” of the order parameter to fluctuations. It is instructive to view this equation as an analog of Newton’s second law of motion, $a = (1/m)F$, with $\phi = (\partial f / \partial |\psi|)$ playing the role of a generalized force that pulls ψ back to equilibrium. We account for fluctuations only in the *magnitude* of the order parameter, since there is no energy cost associated with (mean-field) *phase* fluctuations in the superconducting state.

In our pulse-echo experiment, fluctuations $\delta\psi$ are caused by the oscillating strain field ϵ . For small strains, and hence small fluctuations, there is a small restoring force $\delta\phi = (\partial\phi / \partial |\psi|)|\delta\psi| + (\partial\phi / \partial \epsilon)\delta\epsilon$. We define $Z \equiv \partial\phi / \partial \epsilon = \partial^2 f / \partial \epsilon \partial |\psi|$ and $Y \equiv \partial\phi / \partial |\psi| = \partial^2 f / \partial |\psi|^2$ to obtain an equation of motion for small order-parameter fluctuations $\delta\psi$ induced by a time varying strain $\delta\epsilon$:

$$\frac{\partial|\delta\psi|}{\partial t} = -\tau^{-1}|\delta\psi| - \xi Z \delta\epsilon. \quad (\text{B3})$$

Here $\tau^{-1} = \xi Y$ is the damping rate for order-parameter fluctuations, and the second term can be thought of as a forcing function.

For an oscillating strain, we make the ansatz $\delta\epsilon = \epsilon_0 e^{-i\omega t}$ and $|\delta\psi| = |\delta\psi|(\omega) e^{-i\omega t}$. The equation of motion then gives

$$|\delta\psi(\omega)| = \frac{Z\epsilon_0}{Y} \frac{1}{i\omega\tau - 1}. \quad (\text{B4})$$

We can then use $\sigma = \partial f / \partial \epsilon$ to find the stress response to the strain, $\sigma = \sigma_0 e^{-i\omega t}$. The work done (and hence energy lost) per unit time is given by $\omega \text{Im}(\sigma_0) \text{Re}(\epsilon_0)$, which gives for the attenuation

$$\alpha_{\text{LK}} = \frac{\omega^2 \tau^2}{1 + \omega^2 \tau^2} 4\xi \gamma^2 |\psi_0|^2 \epsilon_0^2. \quad (\text{B5})$$

The only two temperature-dependent terms in this equation are the timescale τ and the equilibrium order parameter ψ_0 . In the small strain approximation these become $\tau = (1/\xi Y) \approx [1/\xi \alpha_0(T_c - T)]$ and $|\psi_0| \approx \sqrt{[\alpha_0(T_c - T)/\beta]}$. τ diverges as the system approaches T_c [33], leading to a large attenuation near T_c . The order parameter ψ_0 itself, however, is zero at T_c , driving the attenuation to zero at T_c . The combination of these two factors leads to a peak in the attenuation slightly below T_c .

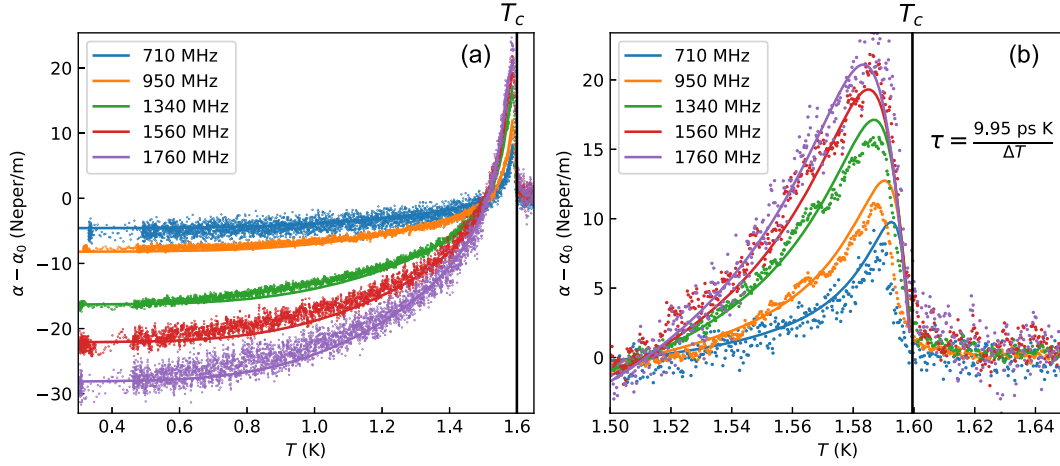


FIG. 7. c_{33} sound attenuation at zero pressure at different frequencies, along with fits to the model described in the Landau-Khalatnikov damping section. (a) Data from 0.3 to 1.65 K. Solid lines correspond to the fits. (b) Data closer to T_c , showing the LK peak in greater detail. The fit described by Eq. (B5) models the data very well, with just three fit parameters used to fit all the data we measure.

Using the temperature dependence for τ and $|\psi_0|$, we solve $\partial\alpha_{LK}/\partial T = 0$ to obtain the position of the attenuation peak. This gives us $T_c - T_{\text{peak}} = \Delta T_{\text{peak}} = \omega/2\alpha_0\xi$. We insert this value back in Eq. (B5) to find the peak attenuation:

$$\alpha_{\text{peak}} = \omega \frac{\gamma^2}{\beta} \epsilon_0^2 = \omega \Delta c \epsilon_0^2. \quad (\text{B6})$$

α_{peak} is linear in frequency with a magnitude that depends only on the size of the elastic modulus jump Δc . We add to the attenuation from Eq. (B5) a gap attenuation $\alpha_{\text{gap}} = \alpha_N f(\Delta(T))$ where α_N is the normal state attenuation, f is the Fermi-Dirac distribution function, and $\Delta(T)$ is the magnitude of the superconducting gap [29] to obtain a total attenuation. We fit this total attenuation to the data in Fig. 7.

APPENDIX C: ORIGIN OF DOMINANT INTERBAND PAIRING

To understand the origin of a dominant interband pairing and the corresponding reduced phase stiffness, we consider a simple model motivated by ferromagnetic spin fluctuations in UTe_2 . A key justification for this model is that density functional theory calculations find that the dominant magnetic interaction is a ferromagnetic interaction between the f electrons on the two U atoms within the unit cell [14]. These ferromagnet correlations are consistent both with neutron observations of spin fluctuations at ambient pressure [36] and with the magnetic order observed under pressure [37]. As shown in more detail below, these ferromagnetic interactions drive spin-triplet superconducting pairing [38] but with a pairing interaction that is largest for interband pairing. As recently shown by Samokhin [41], this dominant interband pairing regime allows a dominant interband superconducting state with a finite T_{c5} that is driven by a Josephson-like coupling to more usual intraband pairing. Importantly, as shown in

Ref. [41] and described in more detail below, this interband pairing state is much more susceptible to phase fluctuations than the more usual intraband Cooper pairs.

More specifically, we consider a model for itinerant f electrons on the two U sites within the unit cell. For clarity, we do not include the Te electrons and also do not include spin-orbit coupling (these will not change the qualitative arguments). The single-particle Hamiltonian that describes these fermions is

$$H = \epsilon_0(k) + f_x(k)\tau_x + f_y(k)\tau_y, \quad (\text{C1})$$

where the τ operators are Pauli matrices that operate in sublattice space. We take $f_x(k) = t_1$ and $f_y(k) = t_2 \sin(k_z/2) \cos(k_x/2) \cos(k_y/2)$. An important parameter in what follows is the ratio $\epsilon = t_2/t_1$. This parameter dictates the relative size of interband pairing to intraband pairing interactions. If it is small, the interband pairing interaction dominates. Fits to the observed Fermi surface at ambient pressure give $\epsilon \ll 1$ ($\epsilon \approx 6 \times 10^{-3}$) [18] and it is physically reasonable that such an inequality also applies at the pressures relevant $P \approx P^*$ here since t_1 is a nearest neighbor hopping and t_2 is a longer-range hopping.

For these f -electron fermions, the intraunit cell ferromagnetic interaction is given by

$$H_M = -|J| \sum_i \vec{S}_{i,1} \cdot \vec{S}_{i,2}, \quad (\text{C2})$$

where $\vec{S}_{i,j} = c_{i,j,s}^\dagger \vec{\sigma}_{s,s'} c_{i,j,s'}$, where $c_{i,j,s}^\dagger$ creates an f electron in unit cell i , U position j within the unit cell, and spin s . This leads to the following pairing interaction:

$$S_C = -\frac{|J|}{V} \int_{k,p,j} (\psi_k^\dagger \tau_y \vec{\sigma}_j i \sigma_y \psi_{-k}) (\psi_{-p}^\dagger \tau_y \vec{\sigma}_j i \sigma_y \psi_p), \quad (\text{C3})$$

where $\psi_k = (c_{1,k,\uparrow}, c_{1,k,\downarrow}, c_{2,k,\uparrow}, c_{2,k,\downarrow})^T$ and V is volume. This pairing interaction implies that a spin-triplet sublattice-singlet pairing state, characterized by $\tau_y \sigma_i (i \sigma_y)$, is stabilized

by these ferromagnetic fluctuations [38]. While describing the pairing as a spin-triplet sublattice-singlet provides a convenient real-space description of superconducting pairing, superconductivity is more naturally described in band (or momentum) space. In general, any real-space Cooper pairing will decompose into usual intraband Cooper pairs together with interband Cooper pairs. Typically, it is assumed that the relevant physics is captured by the intraband Cooper pairing, for which a usual weak-coupling superconducting state naturally emerges [50]. However, this implicitly assumes that the interband pairing interaction can be largely neglected. As pointed out by Samokhin [41], when the interband pairing interaction is sufficiently large, a dominant interband pairing state can emerge. This state is in part stabilized by a Josephson-like coupling between the interband and intraband Cooper pairs. Such a Josephson-like coupling appears because the interband and intraband pairing channels all share the same symmetry.

To see how a predominantly interband pairing state can emerge from Eqs. (C1) and (C3), we express Eq. (C3) in the band basis. The eigenstates in the band basis are found by applying the following unitary transformation to Eq. (C1),

$$U_k = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -e^{-i\phi_k} \\ e^{i\phi_k} & 1 \end{bmatrix}, \quad (C4)$$

where $e^{i\phi_k} = [f_x(k) + if_y(k)]/f(k)$ and $f(k) = \sqrt{f_x^2(k) + f_y^2(k)}$. The consequence of this transformation is to convert the leftmost operator τ_y in Eq. (C3) to

$$U_k \tau_y U_{-k}^T = \begin{bmatrix} -\sin \phi_k & -ie^{-i\phi_k} \cos \phi_k \\ ie^{i\phi_k} \cos \phi_k & \sin \phi_k \end{bmatrix}. \quad (C5)$$

In the above expression, the diagonal elements describe the usual intraband Cooper pairs while the off-diagonal elements describe the interband pairing. Since $\sin \phi_k \approx \epsilon \cos \phi_k$, this implies that the interaction for the interband pairing is parametrically larger than that for the intraband pairing. Following Ref. [41], Eq. (C5) can be reexpressed explicitly in the band basis as

$$S_C = \frac{1}{2V} \sum_{k,k'} \sum_{n_i, s_i} V_{s_1 s_2 s_3 s_4}^{n_1 n_2 n_3 n_4}(k, k') \\ \times a_{k+q/2, n_1, s_1}^\dagger \tilde{a}_{k-q/2, n_2, s_2}^\dagger \tilde{a}_{k'-q/2, n_3, s_3} a_{k'+q/2, n_4, s_4}, \quad (C6)$$

where $\tilde{a}_{k, ns}^\dagger = K a_{k, ns}^\dagger K^{-1}$ is time reversal (K) partner of $a_{k, ns}^\dagger$. We find that the intraband gap functions are usual odd-parity spin-triplet state with a momentum dependence proportional to $\sin(k_z/2) \cos(k_x/2) \cos(k_y/2)$. Furthermore, the interband gap function is also spin-triplet, is antisymmetric under the interchange of the two bands, and predominantly momentum independent. Since the symmetry of all three order parameters is odd parity, it is natural

to ask how this appears in the interband pairing state (which is momentum independent). This can be understood from Eq. (C1) in the limit $t_2 = 0$. In this limit, the two bands that emerge are formed from bonding and antibonding states of the two U atoms. Since the parity symmetry operation exchanges the two U atoms in the unit cell, these bonding and antibonding states are of opposite parity. Consequently, the momentum-independent interband order is odd parity since it pairs fermion states with opposite parity.

To understand the hierarchy of the transition temperatures of the possible pairing states, it is useful to find effective interactions $V^{n_1 n_2 n_3 n_4}$ in powers of ϵ . In our case, we find that the intraband pairing (with order parameters ψ_1 and ψ_2) $V^{ijij} \propto |J|\epsilon_2$, that the interband pairing (with order parameter ψ_m) terms $V_{1212} \propto |J|$, and that the Josephson coupling between the intraband and interband pairs $V^{i112} \propto |J|\epsilon$. Following Ref. [41], this implies that the usual intraband pairing, for which $(\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_m) \approx (1, 1, \epsilon)$, and the dominant interband pairing state, for which $(\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_m) \approx (\epsilon, \epsilon, 1)$, both have a T_c that is of order ϵ^2 . The pairing state that is stabilized then depends upon additional details of the relevant electronic states. At ambient pressure, where there is still some debate about the essential fermiology [51], this appears to be reasonably well understood, and the pairing state (without any magnetic fields applied) is consistent with a usual intraband pairing state. However, for pressures $P \approx P^*$, the essential fermiology is much less well understood. Here we suggest that the dominant interband pairing state has the highest T_c .

To determine the phase stiffness, the Ginzburg free energy to quadratic order must be found. It is possible to do so using standard techniques [41], with the result

$$F_2 = \vec{\psi}^\dagger \hat{A} \vec{\psi} + \kappa_1 |\vec{D}\psi_1|^2 + \kappa_2 |\vec{D}\psi_2|^2 + \tilde{\kappa} |\vec{D}\psi_m|^2, \quad (C7)$$

where $\hat{A}$ describes the order-parameter coupling originating from $V_{n_1 n_2 n_3 n_4}$ (the detailed form of $\hat{A}$ is given in Ref. [41]), $\vec{\eta} = (\psi_1, \psi_2, \psi_m)^T$, κ_i are the usual coefficients for intraband Cooper pairs (below denoted a κ_{usual}), and $\tilde{\kappa}$ is the corresponding coefficient for interband Cooper pairs. Samokhin showed that $\tilde{\kappa}/\kappa_i \approx T_c^2/t^2 \ll 1$. Restricting ourselves to the interband pairing state (which here is the eigenstate of $\hat{A}$ with the largest T_c), $(\psi_1, \psi_2, \psi_m) \approx \psi(\epsilon, \epsilon, 1)$, then the effective coefficient for this state is given by

$$\kappa_{\text{eff}} \approx \epsilon^2 \kappa_{\text{usual}} + \tilde{\kappa} \ll \kappa_{\text{usual}}, \quad (C8)$$

revealing that the dominant interband pairing state is much more susceptible to fluctuations. Note that we have assumed the T_c for the predominantly intraband pairing states are much lower than that for the predominantly interband pairing state, so that their contribution to the phase stiffness can be ignored.

APPENDIX D: GINZBURG NUMBER AT AMBIENT PRESSURE

At ambient pressure, where the broadening due to fluctuations is negligible, there is still a contribution to the width of T_c from impurity and/or inhomogeneous strain broadening. Thus we cannot use the width of the transition alone as a reliable estimate of the Ginzburg number at ambient pressure. In this Appendix, we estimate Gi from ambient-pressure coherence length, penetration depth, and specific heat data. We make three independent estimates of Gi and show that they agree to within an order of magnitude. Note that these rough estimates of Gi are intended only as a point of comparison between the SC1 and SC2 phases.

From the penetration depth data of Ishihara *et al.* [52], we have

$$\begin{aligned}\lambda_a(0) &= 1420 \text{ nm}, \\ \lambda_b(0) &= 710 \text{ nm}, \\ \lambda_c(0) &= 2750 \text{ nm}.\end{aligned}\quad (\text{D1})$$

From the STM data of Sharma *et al.* [53], the a axis and bc plane coherence lengths are found to be

$$\begin{aligned}\xi_a(0) &= 12 \text{ nm}, \\ \xi_{bc}(0) &= 4 \text{ nm}.\end{aligned}\quad (\text{D2})$$

We make another independent estimate of the coherence length using the upper critical field [35,54]:

$$\begin{aligned}\xi(0) &= \sqrt{\frac{\Phi_0}{2\pi\mu_0 H_{c2}(T=0)}} \\ &= 4 \text{ nm}.\end{aligned}\quad (\text{D3})$$

First, we use the formula from Brandt [23] to estimate the Ginzburg number from the zero temperature penetration depth and coherence length:

$$Gi = \frac{1}{2} \left(\frac{k_B T_c \mu_0}{4\pi \xi(0)^3 B_c(0)^2} \right)^2. \quad (\text{D4})$$

Here B_c is the thermodynamic critical field, defined as

$$B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}. \quad (\text{D5})$$

Since the $\lambda(0)$ and $\xi(0)$ of Eqs. (D2) and (D1) are anisotropic, we use their geometric mean to estimate Gi . This is motivated by how Gi is derived by solving the fluctuation contributions from Ginzburg-Landau theory [24]. $\xi(0)$ as derived from Eq. (D3) is of the same order of magnitude and so does not change our conclusions. We get

$$Gi \text{ (from } \lambda \text{ and } \xi) = 4.6 \times 10^{-5}. \quad (\text{D6})$$

The Ginzburg number can also be expressed in terms of the specific heat jump at T_c and the coherence length, which can

be shown to be precisely equal to the Ginzburg number of Eq. (D4) [24]:

$$Gi = \frac{1}{2} \left(\frac{2k_B}{\Delta C \xi^3} \right)^2. \quad (\text{D7})$$

Using the specific heat data of Ref. [17], we estimate $\Delta C/T = 0.2 \text{ J mol}^{-1} \text{ K}^2$. This gives for the Ginzburg number:

$$Gi \text{ (from } \Delta C/T \text{ and } \xi) = 0.8 \times 10^{-5}. \quad (\text{D8})$$

APPENDIX E: KINETIC INDUCTANCE OF UTe_2

Kinetic inductance detectors (KIDs) are highly sensitive single-photon detectors widely used in astrophysical applications. These devices rely on the kinetic inductance of a superconducting film, which sets their sensitivity and scales inversely with the superfluid phase stiffness. In conventional KID materials, such as granular aluminum and NbTiN, low phase stiffness is achieved by introducing disorder into an otherwise stiff superconductor, suppressing the superfluid fraction, and enhancing the kinetic inductance. These high kinetic inductance superconductors are used for small-footprint, high-impedance qubits.

In this Appendix, we compute the kinetic inductance of a thin film of UTe_2 in its pressure-induced SC2 state, which could potentially be stabilized outside of a pressure cell using epitaxial strain. We find that UTe_2 in the SC2 state has a kinetic inductance that is comparable to that of state-of-the-art disordered s -wave superconductors, but in this case it arises intrinsically, without the need for disorder.

We use the known value for the heat capacity jump [13] and the value we calculate for Gi at $P = 0.77 \text{ GPa}$ in Eq. (4) to obtain the phase stiffness κ_0 as

$$\kappa_0 = 1.0 \times 10^{-14} \text{ J/m}. \quad (\text{E1})$$

A thin film made of a superconductor with a phase stiffness κ_0 has a kinetic inductance L_K given by [55]

$$L_K = \frac{1}{8} \left(\frac{\hbar^2}{e^2 \kappa_0^2} \right) \left(\frac{1}{t} \right), \quad (\text{E2})$$

where $\hbar$ is the reduced Planck's constant, e is the electronic charge, and t is the thickness of the superconducting film. For a 3-nm-thick UTe_2 film stabilized in its SC2 state with $Gi = 0.6$, we obtain a kinetic inductance of $1.8 \text{ nH}/\square$. This intrinsic kinetic inductance is comparable to the high kinetic inductance materials used today [48], and is obtained without the need for any disorder.

APPENDIX F: COMPARISON WITH SPECIFIC HEAT

A central point of this work is that the critical fluctuations we observe should also be visible in other thermodynamic

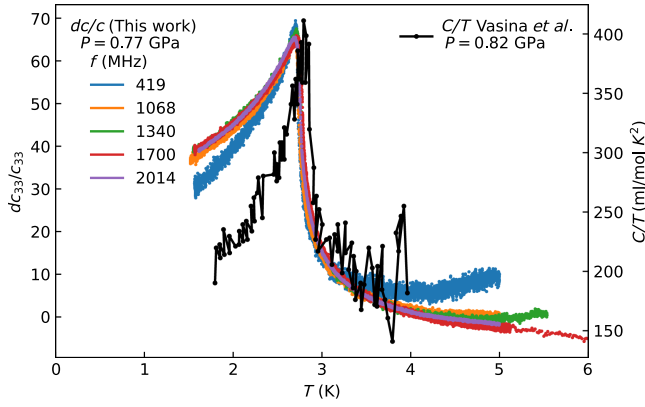


FIG. 8. Fluctuations in the specific heat and c_{33} elastic modulus. We show c_{33} measurements at different frequencies (on an inverted scale) at 0.77 GPa plotted with specific heat (C/T) data from Vasina *et al.* at 0.82 GPa [35]. Fluctuations in both the data are consistent, as is expected of critical fluctuations.

quantities such as the specific heat. In Fig. 8, we plot our data against recent quantitative specific heat measurements [35]. The data overlap well for $T > T_c$, showing consistent fluctuations. For $T < T_c$ the strain-order parameter coupling causes the data to deviate (i.e., the shape below T_c is not universal).

If the broadening at T_{c2} were extrinsic, both measurements would have to experience *exactly* the same pressure distribution, both quantitatively and qualitatively. This is extremely unlikely: The sample sizes used for ultrasound (<0.6 mg) and quantitative heat capacity (23.73 mg [35]) are vastly different, and so are the pressure media (methanol-ethanol and fluorinert [35], respectively). On the other hand, the overlap is exactly what is expected if the fluctuations were caused by thermodynamic criticality.

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Vanishing Phase Stiffness and Fluctuation-Dominated Superconductivity in UTe_2

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PRESSURE HOMOGENIETY

In this section, we quantitatively analyze the broadening observed at T_{c2} for $P = 0.77$ GPa to show that it is an intrinsic effect that cannot be explained by pressure inhomogeneity. We indicate the apparent “width” of T_{c2} with a blue arrow in Figure 1a, extending from $T = 2.7$ K to $T = 3.1$ K. If we assumed that the “intrinsic” T_c was sharp, then the pressure inhomogeneity required to produce such a width in T_{c2} is $\Delta P = 0.21$ GPa, which we indicate with a gray bar in Figure 1c, with T_{c2} bounded by the blue stars. Such a spread in pressure would correspond to a spread of T_c ’s at T_{c1} from $T = 0.6$ K to $T = 0.8$ K—indicated by green stars in Figure 1c. We represent this expected spread as a green arrow in the insets of Figure 1a and b, which show a zoomed in view of the T_{c1} transition. Experimentally, however, we measure a much sharper transition ($\Delta T = 30$ mK), indicating minimal pressure broadening $\Delta P < 0.03$ GPa.

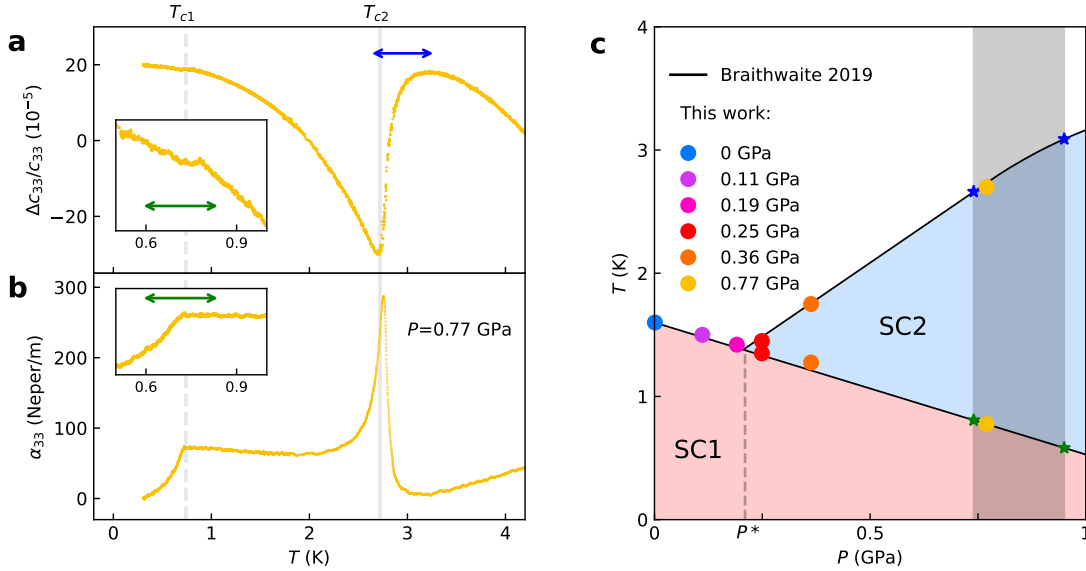


FIG. 1. **A quantitative analysis of pressure inhomogeneity.** The blue arrow in (a) represents the width of the SC2 transition we see at 0.77 GPa — the data with the largest fluctuation region. The pressure inhomogeneity required to produce such a broadening is represented by the gray bar in the phase diagram of (c), with blue stars representing the upper and lower extent of the transition. This inhomogeneity would lead to a transition bounded by the green stars on the SC1 phase boundary. When plotted against the data as the green arrow in the insets of (a) and (b), the true width of the SC1 transition is seen to be much narrower (in particular, it is easy to see that there is no precursor decrease in α_{33} above T_{c2}), implying that the broadening at SC2 is an intrinsic effect.

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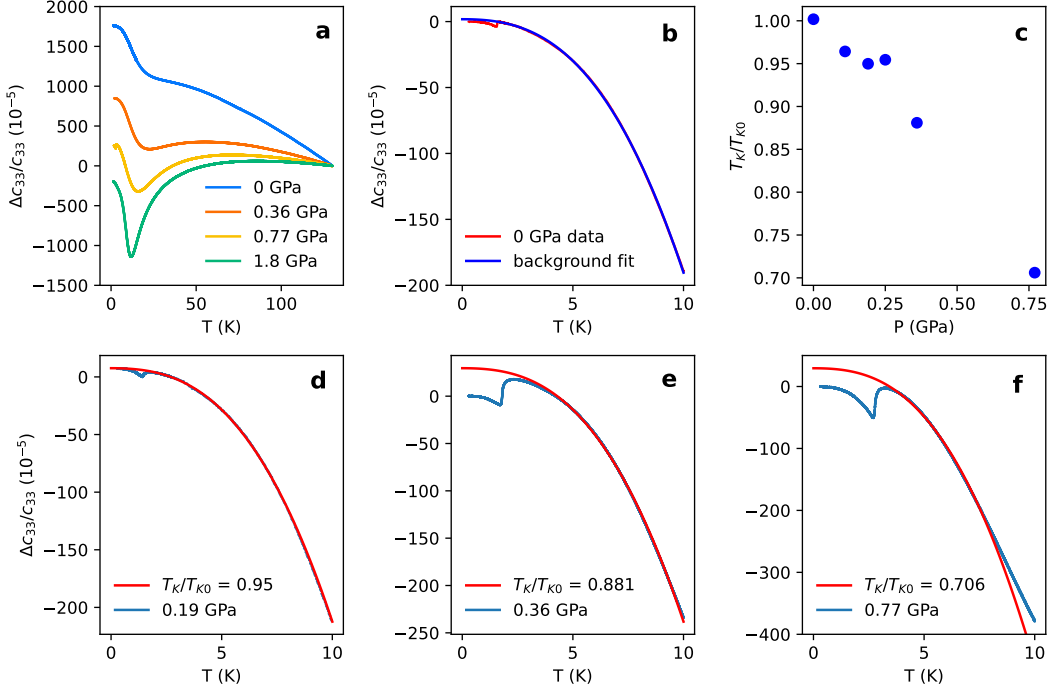


FIG. 2. Background subtraction procedure. a) Raw c_{33} elastic modulus from $T = 2$ to 130 K. b) A polynomial fit to the 0 GPa data for $T > T_c$. c) A plot of the Kondo temperature (extracted from background fits) as a function of pressure. d,e,f) Background fits at three representative pressures. A single variable — the Kondo temperature — is used as a fitting parameter. A deviation of the fit from the data is observed at 0.77 GPa for $T > 8$ K due to the proximity of the minima in c_{33} to the fitting range (the minimum is clearly observable near ≈ 20 K in panel a).

BACKGROUND SUBTRACTION

Figure 2a shows the raw c_{33} data as a function of temperature up to 130 K. These data resemble what is seen for other heavy fermion compounds [1, 2] and are influenced by two factors—a conventional lattice stiffening that monotonically increases c_{33} down to low temperatures, and a Kondo softening that produces the characteristic “dip” at ≈ 20 K, which can be seen most clearly in the 1.8 GPa data. We observe two changes in these curves as a function of increasing pressure: first, the Kondo effect contribution to the elastic constants grows, and second, the temperature at which the minimum occurs decreases.

At temperatures less than 10 K we expect the lattice hardening to have flattened out [3], and so the primary energy scale that governs the background is the Kondo energy scale T_K . In Figure 2b, we fit the c_{33} data at zero pressure to obtain a background. We don’t observe any precursor fluctuations at this pressure, so an accurate background is obtained by fitting a polynomial $f(T)$ to the data from just above T_C up to 10 K.

As the data is governed by a single pressure-dependent energy scale T_K , the behavior at higher pressures can be captured by a simple rescaling of the original polynomial, $f(T) \rightarrow f(\lambda T)$. This scaling reflects a change in the Kondo energy, with $\lambda = T_K/T_{K0}$, where T_{K0} is the zero-pressure Kondo energy. We fit the background to all other pressures by varying only the scaling factor λ . We use data between 4 and 8 K to obtain the fits, as this data is far enough from T_C to not be affected by superconducting fluctuations. These fits are shown in Figure 2d, e and f. Figure 2c shows T_K/T_{K0} obtained from fitting as function of pressure—we see that it smoothly decreases as a function of increasing pressure, as is expected from the qualitative behavior of the high temperature data in Figure 2a and is consistent with other studies of the Kondo energy scale in UTe_2 [4]. Note that at 0.77 GPa, the Kondo temperature scale is low enough that it starts to change the structure of the data at higher temperatures—this causes the background to deviate above ≈ 8 K.

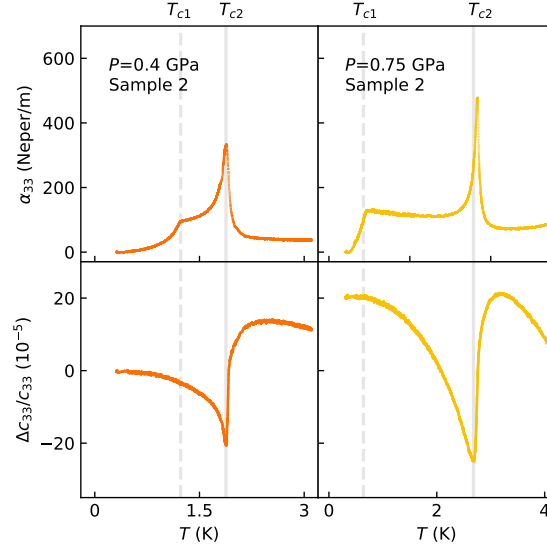


FIG. 3. Elastic modulus (c_{33}) and attenuation (α_{33}) data at two different pressures on sample 2. We observe features similar to those observed in sample 1 in the main text.

SAMPLE 2 DATA

To confirm the reproducibility of the physics presented in the main text, we measure a second sample in the pressure range at which we see large precursor softening in c_{33} (Figure 3). The data is similar to that measured in sample 1, with all salient features still present:

- A peak in α_{33} at T_{c2} that increases with pressure, with a larger attenuation in the SC2 state than the normal state at 0.75 GPa that increases with decreasing temperature.
- A drop in attenuation at T_{c1} .
- Large precursor softening in c_{33} with a diverging contribution that increases with pressure.

ELASTIC MODULI IN THE PRESENCE OF GAUSSIAN FLUCTUATIONS

Saddle point solution

For a system close to a phase transition, we write a Landau Ginzburg Hamiltonian as a function of the (spatially non-uniform) order parameter $\vec{\psi}(\vec{x})$. For brevity, we drop the vector superscript in the following analysis, but our analysis is generally applicable to multi-dimensional order parameters.

$$\mathcal{H}[\psi(x)] = \int d^3x \left[\alpha |\psi(x)|^2 + \frac{\beta}{2} |\psi(x)|^4 + \frac{1}{2} K |\nabla \psi(x)|^2 - \gamma \epsilon_{zz} |\psi(x)|^2 \right]. \quad (1)$$

We define $\alpha = \alpha_0(T - T_c)$, and β is a constant to first order in temperature. $\gamma \equiv \alpha_0 \frac{dT_c}{d\epsilon_{zz}}$ is the strain-superconductivity coupling constant, and K is a stiffness constant for variations in the order parameter. Note that K can be anisotropic in principle, requiring a different stiffness for each vector component of $\nabla \psi$. Since this does not change the physics of the problem, however, we assume an isotropic K . Defining $\alpha' = \alpha - \gamma \epsilon_{zz}$, we can combine the terms quadratic in $\psi(x)$ to write the Hamiltonian as

$$\mathcal{H}[\psi(x)] = \int d^3x \left[\alpha' |\psi(x)|^2 + \frac{\beta}{2} |\psi(x)|^4 + \frac{1}{2} K |\nabla \psi(x)|^2 \right]. \quad (2)$$

The partition function for such a Hamiltonian is:

$$Z = \text{tr} \left[e^{\mathcal{H}/k_B T} \right] = \int \mathcal{D}\psi(x) \exp(\mathcal{H}[\psi(x)]/k_B T). \quad (3)$$

Here $\int \mathcal{D}\psi(x)$ denotes an integral over all the possible configurations of the order parameter $\psi(x)$

We first solve this in the mean-field or saddle-point approximation, i.e. assuming that $\psi(x)$ equals a constant $\bar{\psi}$:

$$\mathcal{H}[\bar{\psi}] = V \left(\alpha' |\bar{\psi}|^2 + \frac{\beta}{2} |\bar{\psi}|^4 \right). \quad (4)$$

The free energy for such a Hamiltonian is given by:

$$F = -k_B T \ln Z = -k_B T \ln \left(e^{-\mathcal{H}[\bar{\psi}]/k_B T} \right) = V \left(\alpha' |\bar{\psi}|^2 + \frac{\beta}{2} |\bar{\psi}|^4 \right) \Big|_{\min}. \quad (5)$$

This has the minima

$$|\bar{\psi}|^2 = \begin{cases} 0 & \text{for } \alpha' > 0 \\ -\frac{\alpha'}{\beta} & \text{for } \alpha' < 0. \end{cases}$$

This gives us for the free energy density $f = F/V$:

$$f = \begin{cases} 0 & \text{for } \alpha' > 0 \\ -\frac{\alpha'^2}{2\beta} & \text{for } \alpha' < 0 \end{cases} \quad (6)$$

We can use this to find the specific heat in the mean-field approximation:

$$C_{\text{MF}} = -T \frac{\partial^2 f}{\partial T^2} = \begin{cases} 0 & \alpha' > 0 \\ -T \frac{\partial^2}{\partial T^2} \left(-\frac{\alpha_0^2 (T - T_c)^2}{2\beta} \right) & \alpha' < 0 \end{cases} \quad (7)$$

$$\Delta C_{\text{MF}} = \frac{\alpha_0^2 T_c}{\beta}. \quad (8)$$

Here ΔC_{MF} is the mean-field specific heat jump at T_c . The elastic constant jump is obtained by a similar calculation:

$$c_{33,\text{MF}} = \frac{\partial^2 f}{\partial \epsilon_{zz}^2} = \begin{cases} 0 & \alpha' > 0 \\ \frac{\partial^2}{\partial \epsilon_{zz}^2} \left(-\frac{\gamma^2 \epsilon_{zz}^2}{2\beta} \right) & \alpha' < 0 \end{cases} \quad (9)$$

$$\Delta c_{33,\text{MF}} = -\frac{\gamma^2}{\beta}. \quad (10)$$

Since $\gamma = \alpha_0 \frac{dT_c}{d\epsilon_{zz}}$, we can combine [Equation 8](#) and [Equation 10](#) to obtain the familiar Ehrenfest's relation:

$$\Delta c_{33,\text{SP}} = -\left(\frac{dT_c}{d\epsilon_{zz}} \right)^2 \frac{\Delta C_{\text{SP}}}{T_c}. \quad (11)$$

Fluctuation terms

We now allow for small spatial variations of the order parameter, perturbing around the mean-field solution for $\bar{\psi}$:

$$\psi(x) = \bar{\psi} + \phi_t(x) \hat{\epsilon}_t + \phi_l(x) \hat{\epsilon}_l. \quad (12)$$

Here $\phi_t(x) \hat{\epsilon}_t$ represents fluctuations transverse to the order parameter direction and $\phi_l(x) \hat{\epsilon}_l$ longitudinal to it. In principle, $\hat{\epsilon}_t$ can be a multicomponent vector.

We assume the fluctuations are small, and hence perturb around the saddle point to second order. When they become large, our theory is not self consistent anymore, and [Equation 3](#) has to be solved exactly to obtain a free energy. To second order in small fluctuations, we get the Hamiltonian

$$\mathcal{H} = V \left(\alpha' \bar{\psi}^2 + \frac{\beta}{2} \bar{\psi}^4 \right) + \int \frac{d^3x}{2} (K(\nabla \phi_t)^2 + (2\alpha' + 2\beta \bar{\psi}^2) \phi_t^2) + \int \frac{d^3x}{2} (K(\nabla \phi_l)^2 + (2\alpha' + 6\beta \bar{\psi}^2) \phi_l^2). \quad (13)$$

Fluctuation corrections for $T > T_C$

The saddle point order parameter $\bar{\psi}$ is zero for $\alpha' > 0$, and so $\alpha' + 3\beta\bar{\psi}^2 = \alpha' + \beta\bar{\psi}^2 = \alpha'$. The partition function for the perturbed Hamiltonian becomes:

$$Z = \int \mathcal{D}\phi_t \exp\left(-\int \frac{d^3x}{2} (K(\nabla\phi_t)^2 + 2\alpha'\phi_t^2) / k_B T\right) \times \int \mathcal{D}\phi_l \exp\left(-\int \frac{d^3x}{2} (K(\nabla\phi_l)^2 + 2\alpha'\phi_l^2) / k_B T\right). \quad (14)$$

The kernels of the Gaussian integrals are diagonalized by the Fourier transformation

$$\phi(x) = \int_q d^3q \frac{e^{iqx}}{\sqrt{V}} \tilde{\phi}(q). \quad (15)$$

The partition function in the Fourier basis becomes

$$Z = \int \mathcal{D}\tilde{\phi}_t \exp\left(-\int \frac{d^3q}{2(2\pi)^3} \left(\frac{Kq^2 + 2\alpha'}{k_B T}\right) \tilde{\phi}_t^2\right) \times \int \mathcal{D}\tilde{\phi}_l \exp\left(-\int \frac{d^3q}{2(2\pi)^3} \left(\frac{Kq^2 + 2\alpha'}{k_B T}\right) \tilde{\phi}_l^2\right). \quad (16)$$

This is a standard Gaussian integral that can then be solved to obtain the free energy density f [5]:

$$f = -k_B T \frac{\ln(Z)}{V} = k_B T \left(\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \ln\left(\frac{Kq^2 + 2\alpha'}{k_B T}\right) + \frac{n-1}{2} \int \frac{d^3q}{(2\pi)^3} \ln\left(\frac{Kq^2 + 2\alpha'}{k_B T}\right) \right) \quad (17)$$

We assume an n component order parameter, leading to $n-1$ components for the transverse fluctuations ϕ_t . The integrands are equal in the $\alpha' > 0$ case, giving us:

$$f = \frac{nk_B T}{2} \int \frac{d^3q}{(2\pi)^3} \ln\left(\frac{Kq^2 + 2\alpha'}{k_B T}\right). \quad (18)$$

Since the mean-field order parameter is zero for $T > T_c$, this term corresponds the corrections to the free energy coming from spatial fluctuations of the order parameter. We can now calculate the fluctuation corrections to the specific heat as we approach T_c from above, using $\alpha' = \alpha_0(T - T_c) - \gamma\epsilon_{zz}$:

$$C_F|_{T>T_c} = -T \frac{\partial^2 f}{\partial T^2} = 2nk_B T^2 \alpha_0^2 \times \frac{1}{8\pi|K^{3/2}|\sqrt{2\alpha_0(T - T_c)}}. \quad (19)$$

A similar calculation is performed for the fluctuation corrections to the elastic constant:

$$c_{33,F}|_{T>T_c} = \frac{\partial^2 f}{\partial \epsilon_{zz}^2} = -2nk_B T \gamma^2 \times \frac{1}{8\pi|K^{3/2}|\sqrt{2\alpha_0(T - T_c)}} \quad (20)$$

Fluctuation corrections for $T < T_C$

For $T < T_c$, the equilibrium value of the order parameter $|\bar{\psi}|^2 = -\alpha'/\beta$. In this case, the perturbed Hamiltonian of Equation 13 becomes

$$\mathcal{H} = V \left(\alpha' \bar{\psi}^2 + \frac{\beta}{2} \bar{\psi}^4 \right) + \int \frac{d^3x}{2} (K(\nabla\phi_t)^2) + \int \frac{d^3x}{2} (K(\nabla\phi_l)^2 - 4\alpha'\phi_l^2). \quad (21)$$

Note that the energetics of longitudinal (magnitude) and transverse (phase) fluctuations are now different. In particular, there are no temperature dependent terms in the integral for transverse fluctuations, and hence they do not contribute to the specific heat. We can perform Gaussian integrals similar to those in the previous section to obtain the free energy density:

$$f = -\frac{k_B T \ln(Z)}{V} = \alpha' \bar{\psi}^2 + \frac{\beta}{2} \bar{\psi}^4 + k_B T \left(\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \ln(Kq^2 - 4\alpha') + \frac{n-1}{2} \int \frac{d^3q}{(2\pi)^3} \ln(Kq^2) \right). \quad (22)$$

The fluctuation part of the specific heat becomes

$$C_F|_{T < T_c} = -T \frac{\partial^2 f}{\partial T^2} = 8k_b T_c^2 \alpha_0^2 \times \frac{1}{8\pi |K^{3/2}| \sqrt{4\alpha_0(T_c - T)}}. \quad (23)$$

The fluctuation part of the elastic modulus becomes

$$c_{33,F}|_{T < T_c} = \frac{\partial^2 f}{\partial \epsilon_{zz}^2} = -8k_b T_c \gamma^2 \times \frac{1}{8\pi |K^{3/2}| \sqrt{4\alpha_0(T_c - T)}}. \quad (24)$$

We can now write both the saddle point and fluctuation contributions to the specific heat jump:

$$\begin{aligned} C|_{T > T_c} &= C_F|_{T > T_c} = \frac{1}{\sqrt{T - T_c}} \frac{nk_B T_c^2 \alpha_0^{3/2}}{4\sqrt{2\pi} |K^{3/2}|} \\ C|_{T < T_c} &= C_{MF} + C_F|_{T < T_c} = \frac{\alpha_0^2 T_c}{\beta} + \frac{1}{\sqrt{T_c - T}} \frac{k_B T_c^2 \alpha_0^{3/2}}{2\pi |K^{3/2}|}; \end{aligned} \quad (25)$$

and the elastic constant jump:

$$\begin{aligned} c_{33}|_{T > T_c} &= c_{33,F}|_{T > T_c} = \frac{-1}{\sqrt{T - T_c}} \frac{nk_B T_c \gamma^2 \alpha_0^{-1/2}}{4\sqrt{2\pi} |K^{3/2}|} \\ c_{33}|_{T < T_c} &= c_{33,MF} + c_{33,F}|_{T < T_c} = \frac{-\gamma^2}{\beta} + \frac{-1}{\sqrt{T_c - T}} \frac{k_B T_c \gamma^2 \alpha_0^{-1/2}}{2\pi |K^{3/2}|}. \end{aligned} \quad (26)$$

A closer inspection of Equation 25 and Equation 26 shows that the Ehrenfest relation from Equation 11 still holds even for non mean-field physics. The two are just related by a multiplicative constant, and thus although the following section will discuss softening in the elastic modulus, the same fluctuations should be observable in a specific heat measurement.

Ginzburg number

We can re-write Equation 26 in terms of physical observables instead of Landau-Ginzburg parameters:

$$\Delta c_{33} = - \left(\frac{dT_c}{d\epsilon_{zz}} \right)^2 \frac{\Delta C_{MF}}{T_c} \times \begin{cases} \frac{\frac{n}{2\sqrt{2t}} \sqrt{\Delta C_{MF} T_c} \left(\frac{k_B T_c}{2\pi} \right)^2}{1 + \frac{1}{\sqrt{t}} \sqrt{\Delta C_{MF} T_c} \left(\frac{k_B T_c}{2\pi} \right)^2} & T > T_c \\ \left(\frac{k_B T_c}{2\pi} \right)^2 & T < T_c. \end{cases} \quad (27)$$

The jump in elastic modulus can be separated into two parts. First, a mean field jump turns on abruptly when $T < T_c$ at the onset of long range order and is given by $-\left(\frac{dT_c}{d\epsilon_{zz}}\right)^2 \left(\frac{\Delta C_{MF}}{T_c}\right)$. Second, a contribution that diverges as $1/\sqrt{t}$ turns on for both $T < T_c$ and $T > T_c$. We *define* the Ginzburg number Gi to be the reduced temperature at which the fluctuation contribution in the superconducting state equals the mean-field jump:

$$Gi = \frac{\Delta C_{MF} T_c}{\kappa_0^3} \left(\frac{k_B T_c}{2\pi} \right)^2. \quad (28)$$

This definition is consistent with previous treatments of fluctuation phenomena in superconductors [6–8], and it allows us to interpret the Ginzburg number as the width of the fluctuation region in reduced temperature. Written in terms of $\Delta c_{33,MF}$ and Gi , Equation 27 becomes

$$\Delta c_{33} = \Delta c_{33,MF} \times \begin{cases} \sqrt{\frac{Gi}{2t}} & T > T_c \\ \left(1 + \sqrt{\frac{Gi}{t}} \right) & T < T_c. \end{cases} \quad (29)$$

We extract Ginzburg numbers by fitting this model to the data in Figure 4. There are just two free parameters since it is only the fluctuation terms that contribute to the elastic modulus change in the normal state: Gi is obtained by

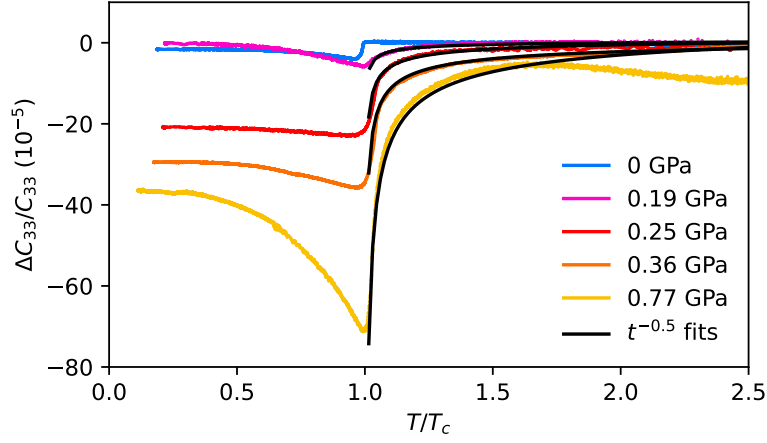


FIG. 4. Background subtracted c_{33} data as a function of T/T_c . The black lines are fits to $at^{-0.5}$, the form the fluctuation contribution assumes in the limit of small fluctuations. The prefactor a , combined with the mean-field c_{33} jump is used to extract a Ginzburg number.

fitting the data for $T > T_c$ to $\sqrt{Gi/2t}$ (Equation 27), and $\Delta c_{33, \text{MF}}$ is obtained from the difference in Δc_{33} at $T/T_c = 0$ and $T/T_c = 2$.

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